Cubic interaction term for Schnabl's solution using Padé approximants

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Abstract

We evaluate the cubic interaction term in the action of open bosonic string field theory for Schnabl's solution written in terms of Bernoulli numbers. This computation provides us with new evidence for the fact that the string field equation of motion is satisfied when it is contracted with the solution itself.

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1. Introduction

A long-standing conjecture by Sen [1, 2] states that, at the stationary point of the tachyon potential on a D25-brane of open bosonic string theory, the negative energy density exactly cancels the tension of the D25-brane. The tachyon potential in Witten's cubic open string field theory [3] has been computed and numerical evidence for Sen's conjecture was given by an approximation scheme called level truncation [4–10]. The action for open bosonic string field theory is

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle\Phi, Q_B\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right], \tag{1.1}$$

where Q_B is the BRST operator of bosonic string theory, $*$ stands for Witten's star product and the inner product $\langle\cdot, \cdot\rangle$ is the standard BPZ inner product. The string field Φ belongs to the full Hilbert space of the first-quantized open string theory. According to Sen's conjecture, the classical open string field equation of motion

$$Q_B\Phi + \Phi * \Phi = 0 \tag{1.2}$$

should admit a Poincaré invariant solution $\Phi \equiv \Psi$ corresponding to the condensation of the open-string tachyon to the vacuum with no D25-branes. This statement means that the energy density of the true vacuum found by solving the equation of motion should be equal to minus

the tension of the D25-brane. Since the energy density of a static configuration is minus the action, Sen's conjecture can be summarized as follows:

$$\frac{1}{g^2}\left[\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi*\Psi\rangle\right] = -\frac{1}{2\pi^2 g^2}. \tag{1.3}$$

The string field equation of motion and Sen's conjecture allow us to fix the kinetic and cubic terms,

$$\frac{\pi^2}{3}\langle\Psi, Q_B\Psi\rangle = -1, \tag{1.4}$$

$$\frac{\pi^2}{3}\langle\Psi, \Psi*\Psi\rangle = 1. \tag{1.5}$$

Recently, Schnabl [11] found an analytic solution to the string field equation of motion, and it was subsequently shown that his solution represents the nonperturbative tachyon vacuum [12–20]. There are two ways of writing Schnabl's analytic solution; the first way is in terms of Bernoulli numbers B_n,

$$\Psi = \sum_{n,p} f_{n,p}(\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p|0\rangle + \sum_{n,p,q} f_{n,p,q}(\mathcal{B}_0 + \mathcal{B}_0^\dagger)(\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p\tilde{c}_q|0\rangle, \tag{1.6}$$

$$f_{n,p} = \frac{1-(-1)^p}{2}\frac{\pi^{-p}}{2^{n-2p+1}}\frac{1}{n!}(-1)^n B_{n-p+1}, \tag{1.7}$$

$$f_{n,p,q} = \frac{1-(-1)^{p+q}}{2}\frac{\pi^{-p-q}}{2^{n-2(p+q)+3}}\frac{1}{n!}(-1)^{n-q} B_{n-p-q+2}, \tag{1.8}$$

whereas the second is in terms of wedge states with ghost insertions,

$$\Psi = \lim_{N\to\infty}\left[\psi_N - \sum_{n=0}^N \partial_n\psi_n\right], \tag{1.9}$$

$$\psi_n = \frac{2}{\pi^2}U_{n+2}^\dagger U_{n+2}\left[(\mathcal{B}_0+\mathcal{B}_0^\dagger)\tilde{c}\left(-\frac{\pi}{4}n\right)\tilde{c}\left(\frac{\pi}{4}n\right)+\frac{\pi}{2}\left(\tilde{c}\left(-\frac{\pi}{4}n\right)+\tilde{c}\left(\frac{\pi}{4}n\right)\right)\right]|0\rangle, \tag{1.10}$$

where ψ_N with $N\to\infty$ is called the phantom term [18–22]. Schnabl's analytic solution was used to prove Sen's conjecture (1.3). Nevertheless, there were subtleties involved in the proof. For instance in a series of two subsequent papers [18, 19], it has been argued that the validity of Schnabl's solution requires that the string field equation of motion be satisfied when it is contracted with the solution itself. This requirement was verified by computing the cubic term (1.5) using Schnabl's solution in terms of wedge states with ghost insertions (1.9). Further numerical evidence for this result was given in [20], where the cubic term was evaluated by using level-truncation computations, i.e. by employing Schnabl's solution written in the usual Virasoro basis. In this work, we use Schnabl's solution written in terms of Bernoulli numbers (1.6) to provide new evidence that the cubic term has the expected value (1.5) predicted from the equation of motion and Sen's conjecture. We evaluate the cubic term using Padé approximants [23, 24], in analogy with the computation of the kinetic term (1.4) performed in [11, 25]. We confirm the expected value of the cubic term required for the string field equation of motion to be satisfied when contracted with the solution itself. This paper is organized as follows. In section 2, we evaluate the cubic term in the action of open bosonic string field theory using Schnabl's solution written in terms of Bernoulli numbers. Here we use Padé approximants to describe how to obtain the expected value of the cubic term. A summary and further directions of exploration are given in section 3. Some details of our calculations such as the evaluation of correlation functions in the $\mathcal{L}_0$ basis and explicit Padé approximants computations are given in the appendices.

2. Evaluating the cubic term

In this section, instead of using the representation of the solution in terms of wedge states with ghost insertions (1.9) or using the solution written in the usual Virasoro basis [20], we evaluate the cubic term using the solution written in terms of Bernoulli numbers (1.6). The computations shown in this section are similar to those in [11, 25], where the kinetic term was evaluated by using the solution written in the $\mathcal{L}_0$ basis, and the expected value (1.4) was reproduced by means of Padé approximants [23, 24]. As described in [11, 25], we start by replacing the solution Ψ with $z^{\mathcal{L}_0}\Psi$ in the $\mathcal{L}_0$-level-truncation scheme, so that states in the $\mathcal{L}_0$ level expansion of the solution will acquire different integer powers of z at different levels. As we are going to see, the parameter z is needed because we need to express the cubic term as a formal power series expansion if we want to use Padé approximants. After doing our calculations, we will simply set $z = 1$.

Let us start with the evaluation of the cubic term as a formal power series expansion in z. Plugging the solution (1.6) into the cubic term and using the correlation functions derived in appendix A we obtain

$$
\begin{aligned}
\langle \Psi, z^{\mathcal{L}_0^\dagger}(z^{\mathcal{L}_0}\Psi) * (z^{\mathcal{L}_0}\Psi)\rangle = {} & \frac{81\sqrt{3}}{8\pi^3}\frac{1}{z^3} + \left[-\frac{81\sqrt{3}}{8\pi^3} + \frac{27}{8\pi^2}\right]\frac{1}{z^2} + \left[\frac{9\sqrt{3}}{4\pi^3} - \frac{3}{2\pi^2} - \frac{\sqrt{3}}{24\pi}\right]\frac{1}{z} \\
& + \left[\frac{1}{180} - \frac{13\pi}{9720\sqrt{3}}\right]z + \left[\frac{1}{270} - \frac{\pi}{1215\sqrt{3}} - \frac{\pi^2}{21870}\right]z^2 \\
& + \left[\frac{5}{4536} + \frac{263\pi}{1224\,720\sqrt{3}} + \frac{71\pi^2}{393\,660} - \frac{59\pi^3}{8266\,860\sqrt{3}}\right]z^3 \\
& + \left[-\frac{1}{5670} + \frac{113\pi}{183\,708\sqrt{3}} + \frac{40\pi^2}{137\,781} - \frac{8\pi^3}{413\,343\sqrt{3}}\right. \\
& \left. - \frac{5\pi^4}{11\,160\,261}\right]z^4 + \cdots.
\end{aligned}
\tag{2.1}
$$

At this point we remark that the most cumbersome of our computations is the evaluation of correlation functions which come from plugging the solution (1.6) into the cubic term; the details of these computations are shown in appendix A. Once the respective correlation functions are computed, in principle, it should be possible to write the series (2.1) to any order in powers of z. Nevertheless, the time it takes to do these calculations increases considerably with every subsequent power of z. Given the formal power series expansion (2.1), we are able to evaluate the cubic term using Padé approximants. We match the power series expansion coefficients of a given rational function $P_{3+M}^N(z)$ with those of the cubic term (2.1). The details of these computations can be found in appendix B. The main result of our work is summarized in table 1. The first column is the definition of the cubic term in the $\mathcal{L}_0$ level truncation. As we can see in the second column, the value of the cubic term computed using Padé approximants converges to the expected value (1.5). We note that the value of the cubic term for n greater than 8 shown in the first column has an oscillating behavior. Let us mention that a series may diverge either by approaching infinity or by oscillating. An example of a divergent series that diverges by going to infinity is the series corresponding to the kinetic term [11, 25]. It seems that in the case of the cubic term, the divergent character of the series is due to its oscillating behavior, which would be interesting to verify by performing higher level computations. Since Padé approximants can deal numerically with divergent series [23, 24], we have shown by explicit computations that our results confirm the expected value of the cubic term (1.5).

Table 1. The Padé approximation for the normalized value of the cubic term $\frac{\pi^2}{3}\langle\Psi, z^{\mathcal{L}_0^\dagger}(z^{\mathcal{L}_0}\Psi)*$ $(z^{\mathcal{L}_0}\Psi)\rangle$ evaluated at $z=1$. The first column is a naive evaluation of the cubic term given by the series (2.1), and the second column is its respective $P_{3+n/2}^{n/2}$ Padé approximation. The label n corresponds to the power of z in the series (2.1). At each stage of our computations we truncate the series up to the order z^{n-3}.

	Naive computation	$P_{3+n/2}^{n/2}$ Padé approximation
$n=0$	1.860 735 02	1.860 735 02
$n=2$	0.962 921 69	0.917 128 84
$n=4$	0.973 217 97	0.976 204 55
$n=6$	0.989 350 43	0.973 969 38
$n=8$	1.005 983 43	1.004 139 34
$n=10$	1.001 709 26	1.005 194 20
$n=12$	0.994 788 28	1.000 215 92
$n=14$	1.004 169 03	1.000 100 61
$n=16$	1.002 231 24	1.000 166 72
$n=18$	0.994 335 56	0.999 978 63
$n=20$	1.009 117 57	0.999 982 42

3. Summary and discussion

We computed the cubic term in the $\mathcal{L}_0$-level-truncation scheme [11, 25], and we provided new evidence for the fact that Schnabl's tachyon solution of open bosonic string field theory is valid in the sense that it solves the equation of motion when it is contracted with the solution itself.

Up to the level that we explored with our computations, it is worth remarking that the series that defines the cubic term (2.1) seems to have an oscillating behavior. This character of the series is in contrast with the character of the series for the case of the kinetic term which does not begin to diverge until higher levels, where computations reveal it starts to go to infinity [11, 25]. In the case of the cubic term, we could perform higher level computations to confirm the oscillating behavior of the series. We hope that the approach used in [25] when applied to the case of the cubic term will help to clarify this issue.

A direct application of the results shown in this paper is related to the study of level-truncation computations in the $\mathcal{L}_0$ basis. In this basis, the analytic solution found by Schnabl was originally obtained by truncating the equation of motion but not the string field, so it would be interesting to analyze the case when we truncate the string field instead of the equation of motion. This analysis should serve us to address some issues, e.g., the computation of the effective tachyon potential in Schnabl's gauge.

A second application would be the extension of our methods to the case of the Berkovits superstring field theory [26]. In this formalism, we already have a solution for the tachyon condensate written in the $\mathcal{L}_0$ basis [27]. Obviously, the next step would be the evaluation of the energy. We hope that Padé approximants will confirm the expected value predicted from D-brane arguments [28].

Acknowledgments

Appendix A. Correlation functions and the cubic term

All correlation functions shown in this appendix are evaluated on the semi-infinite cylinder C_π with circumference π. The relation between correlation functions evaluated on the upper-half plane (UHP) and those evaluated on the semi-infinite cylinder is given in [11], where the conformal map $\arctan z$ is used to map the UHP to the semi-infinite cylinder. Employing the definition of the conformal transformation $\tilde{c}(x) = \cos^2(x)c(\tan x)$ of the c ghost (under the conformal map in the paragraph above) and its anticommutator relation with the operators $\mathcal{B}_0$ and B_1,[2]

$$\{\mathcal{B}_0, \tilde{c}(z)\} = z, \tag{A.1}$$

$$\{B_1, \tilde{c}(z)\} = 1, \tag{A.2}$$

we obtain the following basic correlation functions:

$$\langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle = \sin(x - y)\sin(x - z)\sin(y - z), \tag{A.3}$$

$$\langle \tilde{c}(x)\mathcal{B}_0\tilde{c}(y)\tilde{c}(z)\tilde{c}(w)\rangle = y\langle \tilde{c}(x)\tilde{c}(z)\tilde{c}(w)\rangle - z\langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle + w\langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle, \tag{A.4}$$

$$\langle \tilde{c}(x)\tilde{c}(y)\mathcal{B}_0\tilde{c}(z)\tilde{c}(w)\rangle = z\langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle - w\langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle, \tag{A.5}$$

$$\langle \tilde{c}(x)B_1\tilde{c}(y)\tilde{c}(z)\tilde{c}(w)\rangle = \langle \tilde{c}(x)\tilde{c}(z)\tilde{c}(w)\rangle - \langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle + \langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle, \tag{A.6}$$

$$\langle \tilde{c}(x)\tilde{c}(y)B_1\tilde{c}(z)\tilde{c}(w)\rangle = \langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(w)\rangle - \langle \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle. \tag{A.7}$$

To compute correlation functions involved in the evaluation of the cubic term, the following contour integrals will be very useful:

$$\sigma(a) \equiv \oint \frac{dz}{2\pi i} z^a \sin(2z)$$
$$= \frac{\theta(-a - 2)}{\Gamma(-a)}((-1)^a + 1)(-1)^{\frac{2-a}{2}} 2^{-a-2}, \tag{A.8}$$

$$\varsigma(a) \equiv \oint \frac{dz}{2\pi i} z^a \cos(2z)$$
$$= \frac{\theta(-a - 1)}{\Gamma(-a)}((-1)^a - 1)(-1)^{\frac{1-a}{2}} 2^{-a-2}, \tag{A.9}$$

$$\mathcal{F}(a_1, a_2, a_3, \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3)$$
$$\equiv \oint \frac{dx_1 dx_2 dx_3}{(2\pi i)^3} x_1^{a_1} x_2^{a_2} x_3^{a_3} \langle \tilde{c}(\alpha_1 x_1 + \beta_1)\tilde{c}(\alpha_2 x_2 + \beta_2)\tilde{c}(\alpha_3 x_3 + \beta_3)\rangle$$
$$= \frac{1}{\alpha_1^{a_1+1}\alpha_2^{a_2+1}\alpha_3^{a_3+1}}$$
$$\Big[\delta_{a_3,-1}$$
$$\times \frac{(\sigma(a_1)\sigma(a_2) + \varsigma(a_1)\varsigma(a_2))\sin(2(\beta_1 - \beta_2)) + (\sigma(a_1)\varsigma(a_2) - \varsigma(a_1)\sigma(a_2))\cos(2(\beta_1 - \beta_2))}{4}$$
$$+ \delta_{a_2,-1}$$
$$\times \frac{(\varsigma(a_1)\sigma(a_3) - \sigma(a_1)\varsigma(a_3))\cos(2(\beta_1 - \beta_3)) - (\varsigma(a_1)\varsigma(a_3) + \sigma(a_1)\sigma(a_3))\sin(2(\beta_1 - \beta_3))}{4}$$

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[2] The operators $\mathcal{B}_0$ and $B_1 \equiv \mathcal{B}_{-1}$ are modes of the b ghost which are defined on the semi-infinite cylinder coordinate as follows: $\mathcal{B}_n = \oint \frac{dz}{2\pi i}(1 + z^2)(\arctan z)^{n+1} b(z)$.

$$+ \delta_{a_1,-1}$$

$$\times \frac{(\varsigma(a_2)\varsigma(a_3) + \sigma(a_2)\sigma(a_3))\sin(2(\beta_2 - \beta_3)) + (\sigma(a_2)\varsigma(a_3) - \varsigma(a_2)\sigma(a_3))\cos(2(\beta_2 - \beta_3))}{4}\Bigg],$$

$$(A.10)$$

where $\theta(n)$ is the unit step (Heaviside) function which is defined as follows:

$$\theta(n) = \begin{cases} 0, & \text{if } n < 0 \\ 1, & \text{if } n \geqslant 0. \end{cases} \qquad (A.11)$$

Let us list a few non-trivial correlation functions which involve operators frequently used in the $\mathcal{L}_0$ basis, namely $\hat{\mathcal{L}}^n$ ($\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger$), $\hat{\mathcal{B}}$ ($\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger$), $U_r = \left(\frac{2}{r}\right)^{\mathcal{L}_0}$ and the $\tilde{c}(z)$ ghost:

$$\langle \text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1} U_r^\dagger U_r \tilde{c}(x)\tilde{c}(y)\rangle$$

$$= \oint \frac{\mathrm{d}z_1 \mathrm{d}x_1}{(2\pi\mathrm{i})^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2}$$

$$\times \left\langle \tilde{c}\left(x_1 + \frac{\pi}{2}\right)\tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\right\rangle, \qquad (A.12)$$

$$\langle \text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}U_r^\dagger U_r \tilde{c}(x)\tilde{c}(y)\tilde{c}(z)\rangle$$

$$= -\delta_{p_1,0} \oint \frac{\mathrm{d}z_1}{2\pi\mathrm{i}} \frac{(-2)^{n_1} n_1!}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2}$$

$$\times \left\langle \tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\tilde{c}\left(\frac{4}{z_1 r}z\right)\right\rangle$$

$$+ \oint \frac{\mathrm{d}z_1 \mathrm{d}x_1}{(2\pi\mathrm{i})^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1+n_1-2} \left(\frac{2}{z_1}\right)^{-p_1-2}$$

$$\times \left\langle \tilde{c}\left(x_1 + \frac{\pi}{2}\right)\mathcal{B}_0\tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\tilde{c}\left(\frac{4}{z_1 r}z\right)\right\rangle, \qquad (A.13)$$

$$\langle \text{bpz}(\tilde{c}_{p_1})\text{bpz}(\tilde{c}_{p_2})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}U_r^\dagger U_r \tilde{c}(x)\tilde{c}(y)\rangle$$

$$= -\delta_{p_2,0} \oint \frac{\mathrm{d}z_1 \mathrm{d}x_1}{(2\pi\mathrm{i})^2} \frac{(-2)^{n_1} n_1! x_1^{p_1-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1}$$

$$\times \left\langle \tilde{c}\left(x_1 + \frac{\pi}{2}\right)\tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\right\rangle$$

$$+ \delta_{p_1,0} \oint \frac{\mathrm{d}z_1 \mathrm{d}x_2}{(2\pi\mathrm{i})^2} \frac{(-2)^{n_1} n_1! x_2^{p_2-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1}$$

$$\times \left\langle \tilde{c}\left(x_2 + \frac{\pi}{2}\right)\tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\right\rangle$$

$$+ \oint \frac{\mathrm{d}z_1 \mathrm{d}x_1 \mathrm{d}x_2}{(2\pi\mathrm{i})^3} \frac{(-2)^{n_1} n_1! x_1^{p_1-2} x_2^{p_2-2}}{(z_1-2)^{n_1+1}} \left(\frac{2}{r}\right)^{-p_1-p_2+n_1-1} \left(\frac{2}{z_1}\right)^{-p_1-p_2-1}$$

$$\times \left\langle \tilde{c}\left(x_1 + \frac{\pi}{2}\right)\tilde{c}\left(x_2 + \frac{\pi}{2}\right)\mathcal{B}_0\tilde{c}\left(\frac{4}{z_1 r}x\right)\tilde{c}\left(\frac{4}{z_1 r}y\right)\right\rangle, \qquad (A.14)$$

where the 'bpz' acting on the modes of the $\tilde{c}(z)$ ghost stands for the usual BPZ conjugation which in the $\mathcal{L}_0$ basis is defined as follows:

$$\text{bpz}(\tilde{\phi}_n) = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+h-1}\tilde{\phi}\left(z + \frac{\pi}{2}\right), \qquad (A.15)$$

for any primary field $\tilde{\phi}(z)$ with weight h. The action of the BPZ conjugation on the modes of $\tilde{\phi}(z)$ satisfies the following useful property:

$$U_r^{\dagger-1}\mathrm{bpz}(\tilde{\phi}_n)U_r^\dagger = \left(\frac{2}{r}\right)^{-n}\mathrm{bpz}(\tilde{\phi}_n). \tag{A.16}$$

Here we provide a few intermediate steps regarding the evaluation of the cubic term of the open string field action for Schnabl's solution Ψ expressed in terms of Bernoulli numbers (1.6). As discussed in section 2, in order to apply Padé approximants we must start by replacing the solution Ψ with $z^{\mathcal{L}_0}\Psi$. In the $\mathcal{L}_0$ level expansion different levels will acquire different integer powers of z. Plugging this redefinition of the solution into the cubic term of the action we obtain

$$\langle\Psi, z^{\mathcal{L}_0^\dagger}(z^{\mathcal{L}_0}\Psi)*(z^{\mathcal{L}_0}\Psi)\rangle$$

$$= \sum_{n_1,n_2,n_3,p_1,p_2,p_3} f_{n_1,p_1}f_{n_2,p_2}f_{n_3,p_3}\Delta^{(1)}_{n_1,n_2,n_3,p_1,p_2,p_3}z^{n_1+n_2+n_3-p_1-p_2-p_3}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} f_{n_1,p_1}f_{n_2,p_2}f_{n_3,p_3,p_4}\Delta^{(2)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4}z^{n_1+n_2+n_3+1-p_1-p_2-p_3-p_4}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} f_{n_1,p_1}f_{n_2,p_2,p_3}f_{n_3,p_4}\Delta^{(3)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4}z^{n_1+n_2+n_3+1-p_1-p_2-p_3-p_4}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} f_{n_1,p_1,p_2}f_{n_2,p_3}f_{n_3,p_4}\Delta^{(4)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4}z^{n_1+n_2+n_3+1-p_1-p_2-p_3-p_4}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} f_{n_1,p_1}f_{n_2,p_2,p_3}f_{n_3,p_4,p_5}\Delta^{(5)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5}z^{n_1+n_2+n_3+2-p_1-p_2-p_3-p_4-p_5}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} f_{n_1,p_1,p_2}f_{n_2,p_3}f_{n_3,p_4,p_5}\Delta^{(6)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5}z^{n_1+n_2+n_3+2-p_1-p_2-p_3-p_4-p_5}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} f_{n_1,p_1,p_2}f_{n_2,p_3,p_4}f_{n_3,p_5}\Delta^{(7)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5}z^{n_1+n_2+n_3+2-p_1-p_2-p_3-p_4-p_5}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5,p_6} f_{n_1,p_1,p_2}f_{n_2,p_3,p_4}f_{n_3,p_5,p_6}\Delta^{(8)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5,p_6}$$

$$z^{n_1+n_2+n_3+3-p_1-p_2-p_3-p_4-p_5-p_6}, \tag{A.17}$$

where to simplify notation we have used the following definitions:

$$\Delta^{(1)}_{n_1,n_2,n_3,p_1,p_2,p_3} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_3}|0\rangle, \tag{A.18}$$

$$\Delta^{(2)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\hat{\mathcal{B}}\tilde{c}_{p_3}\tilde{c}_{p_4}|0\rangle, \tag{A.19}$$

$$\Delta^{(3)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\hat{\mathcal{B}}\tilde{c}_{p_2}\tilde{c}_{p_3}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_4}|0\rangle, \tag{A.20}$$

$$\Delta^{(4)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\mathrm{bpz}(\tilde{c}_{p_2})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_3}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_4}|0\rangle, \tag{A.21}$$

$$\Delta^{(5)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\hat{\mathcal{B}}\tilde{c}_{p_2}\tilde{c}_{p_3}|0\rangle * \hat{\mathcal{L}}^{n_3}\hat{\mathcal{B}}\tilde{c}_{p_4}\tilde{c}_{p_5}|0\rangle, \tag{A.22}$$

$$\Delta^{(6)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\mathrm{bpz}(\tilde{c}_{p_2})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_3}|0\rangle * \hat{\mathcal{L}}^{n_3}\hat{\mathcal{B}}\tilde{c}_{p_4}\tilde{c}_{p_5}|0\rangle, \tag{A.23}$$

$$\Delta^{(7)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5} \equiv \langle 0|\mathrm{bpz}(\tilde{c}_{p_1})\mathrm{bpz}(\tilde{c}_{p_2})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}, \hat{\mathcal{L}}^{n_2}\hat{\mathcal{B}}\tilde{c}_{p_3}\tilde{c}_{p_4}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_5}|0\rangle, \tag{A.24}$$

$$\Delta^{(8)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4,p_5,p_6} \equiv \langle 0|\text{bpz}(\tilde{c}_{p_1})\text{bpz}(\tilde{c}_{p_2})\hat{\mathcal{L}}^{n_1}\hat{\mathcal{B}}, \hat{\mathcal{L}}^{n_2}\hat{\mathcal{B}}\tilde{c}_{p_3}\tilde{c}_{p_4}|0\rangle * \hat{\mathcal{L}}^{n_3}\hat{\mathcal{B}}\tilde{c}_{p_5}\tilde{c}_{p_6}|0\rangle \quad \text{(A.25)}$$

for all the correlation functions appearing in the evaluation of the cubic term.

　　All these correlation functions can be readily computed using the results of this appendix. For instance, let us compute the correlator $\langle 0|\text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_3}|0\rangle$ which involves states of the form $\hat{\mathcal{L}}^n\tilde{c}_p|0\rangle$,

$$\langle 0|\text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, \hat{\mathcal{L}}^{n_2}\tilde{c}_{p_2}|0\rangle * \hat{\mathcal{L}}^{n_3}\tilde{c}_{p_3}|0\rangle$$

$$= \frac{(-2)^{n_2+n_3}n_2!n_3!}{(2\pi\mathrm{i})^4}\oint\frac{\mathrm{d}z_2\mathrm{d}z_3\mathrm{d}x_2\mathrm{d}x_3 x_2^{p_2-2}x_3^{p_3-2}}{(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}}$$

$$\times \langle 0|\text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}, U_{z_2}^\dagger U_{z_2}\tilde{c}(x_2)|0\rangle * U_{z_3}^\dagger U_{z_3}\tilde{c}(x_3)|0\rangle$$

$$= \frac{(-2)^{n_2+n_3}n_2!n_3!}{(2\pi\mathrm{i})^4}\oint\frac{\mathrm{d}z_2\mathrm{d}z_3\mathrm{d}x_2\mathrm{d}x_3 x_2^{p_2-2}x_3^{p_3-2}}{(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}}$$

$$\times \left\langle\text{bpz}(\tilde{c}_{p_1})\hat{\mathcal{L}}^{n_1}U_r^\dagger U_r\tilde{c}\left(x_2+\frac{\pi}{4}(z_3-1)\right)\tilde{c}\left(x_3-\frac{\pi}{4}(z_2-1)\right)\right\rangle$$

$$= \frac{(-1)^{n_1+n_2+n_3}2^{2n_1+n_2+n_3-2p_1-4}n_1!n_2!n_3!}{(2\pi\mathrm{i})^3}\oint\frac{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}z_3 z_1^{p_1+2}r^{p_1+2-n_1}}{(z_1-2)^{n_1+1}(z_2-2)^{n_2+1}(z_3-2)^{n_3+1}}$$

$$\times \mathcal{F}\left(p_1-2, p_2-2, p_3-2, 1, \frac{\pi}{2}, \frac{4}{z_1 r}, \frac{\pi(z_3-1)}{z_1 r}, \frac{4}{z_1 r}, \frac{\pi(1-z_2)}{z_1 r}\right), \quad \text{(A.26)}$$

where we have defined $r \equiv z_2 + z_3 - 1$ and used the definition of $\mathcal{F}$ (A.10).

　　Although the expression for the cubic term (A.17) looks complicated, it is actually quite easy to simplify the expression. Using the cyclicity symmetry of the three-vertex

$$\langle A, B*C\rangle = (-1)^{\text{gh}(A)(\text{gh}(B)+\text{gh}(C))}\langle B, C*A\rangle = (-1)^{\text{gh}(C)(\text{gh}(A)+\text{gh}(B))}\langle C, A*B\rangle,$$

and the following star product identities involving the $\mathcal{B}_0$ operator:

$$((\mathcal{B}_0+\mathcal{B}_0^\dagger)\phi_1)*\phi_2 = (\mathcal{B}_0+\mathcal{B}_0^\dagger)(\phi_1*\phi_2) + (-1)^{\text{gh}(\phi_1)}\frac{\pi}{2}\phi_1*B_1\phi_2,$$

$$\phi_1*((\mathcal{B}_0+\mathcal{B}_0^\dagger)\phi_2) = (-1)^{\text{gh}(\phi_1)}(\mathcal{B}_0+\mathcal{B}_0^\dagger)(\phi_1*\phi_2) - (-1)^{\text{gh}(\phi_1)}\frac{\pi}{2}(B_1\phi_1)*\phi_2,$$

$$((\mathcal{B}_0+\mathcal{B}_0^\dagger)\phi_1)*((\mathcal{B}_0+\mathcal{B}_0^\dagger)\phi_2) = -(-1)^{\text{gh}(\phi_1)}\frac{\pi}{2}(\mathcal{B}_0+\mathcal{B}_0^\dagger)B_1(\phi_1*\phi_2)$$

$$+\left(\frac{\pi}{2}\right)^2(B_1\phi_1)*(B_1\phi_2),$$

we obtain the following simplified expression for the cubic term:

$$\langle\Psi, z^{\mathcal{L}_0^\dagger}(z^{\mathcal{L}_0}\Psi)*(z^{\mathcal{L}_0}\Psi)\rangle$$

$$= \sum_{n_1,n_2,n_3,p_1,p_2,p_3}\left[f_{n_1,p_1}f_{n_2,p_2}f_{n_3,p_3} + 3\pi^2 f_{n_1,p_1}f_{n_2,1,p_2}f_{n_3,1,p_3}\right]\Delta^{(1)}_{n_1,n_2,n_3,p_1,p_2,p_3}z^{n_1+n_2+n_3-p_1-p_2-p_3}$$

$$+ \sum_{n_1,n_2,n_3,p_1,p_2,p_3,p_4}\left[3f_{n_1,p_1}f_{n_2,p_2}f_{n_3,p_3,p_4} + \pi^2 f_{n_1,1,p_1}f_{n_2,1,p_2}f_{n_3,p_3,p_4} + 3\pi f_{n_1,p_1}f_{n_2,1,p_2}f_{n_3,p_3,p_4}\right.$$

$$\left.-3\pi f_{n_2,p_2}f_{n_1,1,p_1}f_{n_3,p_3,p_4}\right]\Delta^{(2)}_{n_1,n_2,n_3,p_1,p_2,p_3,p_4}z^{n_1+n_2+n_3+1-p_1-p_2-p_3-p_4}.$$

Therefore, we only need to compute the correlation functions (A.18) and (A.19). As it was already mentioned these correlation functions can be readily computed by using the correlators (A.3)–(A.7), (A.12)–(A.14). We were aided by a computer to perform these calculations.

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Appendix B. Padé approximant computations

Here we shall explain the method to calculate the cubic term based on Padé approximants by computing in detail the normalized value of the cubic term at order $n = 4$, shown in table 1. At this order, we need to consider terms in the series (2.1) up to linear order in z, namely

$$\frac{81\sqrt{3}}{8\pi^3}\frac{1}{z^3} + \left[-\frac{81\sqrt{3}}{8\pi^3} + \frac{27}{8\pi^2}\right]\frac{1}{z^2} + \left[\frac{9\sqrt{3}}{4\pi^3} - \frac{3}{2\pi^2} - \frac{\sqrt{3}}{24\pi}\right]\frac{1}{z} + \left[\frac{1}{180} - \frac{13\pi}{9720\sqrt{3}}\right]z. \quad \text{(B.1)}$$

Using Padé approximants, we express (B.1) as the following rational function:

$$P_{3+2}^2(z) = \frac{1}{z^3}\left[\frac{a_0 + a_1 z + a_2 z^2}{1 + b_1 z + b_2 z^2}\right]. \quad \text{(B.2)}$$

Expanding the right-hand side of (B.2) around $z = 0$, we get up to linear order in z

$$\begin{aligned}
P_{3+2}^2(z) = & \frac{a_0}{z^3} + \frac{a_1 - a_0 b_1}{z^2} + \frac{a_2 - a_1 b_1 + a_0 b_1^2 - a_0 b_2}{z} \\
& + \left(a_1 b_1^2 - a_2 b_1 - a_0 b_1^3 - a_1 b_2 + 2a_0 b_1 b_2\right) \\
& + \left(a_2 b_1^2 - a_1 b_1^3 + a_0 b_1^4 - a_2 b_2 + 2a_1 b_1 b_2 - 3a_0 b_1^2 b_2 + a_0 b_2^2\right)z. \quad \text{(B.3)}
\end{aligned}$$

Equating the coefficients of z^{-3}, z^{-2}, z^{-1}, z^0, z^1 in equations (B.1) and (B.3), we get a system of five algebraic equations for the unknown coefficients a_0, a_1, a_2, b_1 and b_2. Solving these equations we get

$$a_0 = 0.565595624636, \quad \text{(B.4)}$$

$$a_1 = -0.38673808434, \quad \text{(B.5)}$$

$$a_2 = 0.051154789816, \quad \text{(B.6)}$$

$$b_1 = -0.28837113902, \quad \text{(B.7)}$$

$$b_2 = 0.063526545755. \quad \text{(B.8)}$$

Replacing the value of the coefficients (B.4)–(B.8) into the definition of $P_{3+2}^2(z)$ (B.2), and evaluating this at $z = 1$, we get the following normalized value for the cubic term:

$$\frac{\pi^2}{3}P_{3+2}^2(z = 1) = 0.976204550211. \quad \text{(B.9)}$$

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